## Basically, we have three different means to reconstruct phylogenetic trees from sequence data



## Calculating tree likelihoods



For an alignment of four sequences and length $\mathrm{m}=3$ the likelihood is then

$$
\begin{aligned}
L(T)=\prod_{k=1}^{m} L^{(k)}= & 0.005331^{2} \times 0.005331 \\
& =0.000000152
\end{aligned}
$$

or the log-likelihood is

$$
\ln L(T)=\sum_{k=1}^{m} \ln L^{(k)}=-15.7
$$

Now that we know how to evaluate the likelihood of any given tree, we need to ask how to find the ML tree

Heuristic tree search begins with an initial sub-optimal solution (starting tree) obtained either via step-wise addition (or using a distance tree)


Finding the best tree
Evaluate random rearrangements of the starting tree and accept new tree if it improves $P(D \mid M, T)$. Continue until convergence.


Again we have an iterative stochastic process as we have seen in the alignment case

## Resampling methods for assessing the support of a $\left(\mathrm{ML}^{1}\right)$ tree given the data

Rationale: All positions in a sequence, and hence all alignment columns, should have the same evolutionary history. Thus, we can summarize the phylogenetic information in a single tree.


## Resampling methods for assessing the support of a tree given the data

Rationale: All positions in a sequence, and hence all alignment columns, should have the same evolutionary history. Thus, it should in principle not matter which subset of the data I am using for tree reconstruction if the phylogenetic signal is sufficiently strong and indeed consistent.

| Taxon | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S1 | C | G | C | G | C | T | G | T | T |
| S2 | C | G | C | A | C | T | C | T | T |
| S3 | T | G | A | A | C | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |
|  |  |  |  |  |  |  |  |  |  |

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| Taxon | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | C | G | C | G | C | T | G | T | T |
| S2 | C | G | C | A | C | T | C | T | T |
| S3 | T | G | A | A | C | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

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## Resampling methods for assessing the support of a tree given the data

Observation: The phylogenetic signal in the data is apparently not entirely consistent and we would like to have a method to assess the extent of variability.

| Taxon | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | C | G | C | G | C | T | G | T | T |
| S2 | C | G | C | A | C | T | C | T | T |
| S3 | T | G | A | A | C | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |
|  |  |  |  |  | V |  |  |  |  |



## Resampling methods for assessing the support of a tree given the data

Approach 1 - Jackknife: Remove a random subset of alignment columns and re-compute the tree. Typically a $50 \%$ Jackknife analysis is performed.


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| Taxon | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | 6 | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S 1}$ | C | G | C | G | C | T | G | T | T |
| $\mathbf{S 2}$ | C | G | C | A | C | T | C | T | T |
| $\mathbf{S 3}$ | T | G | A | A | C | T | G | C | T |
| $\mathbf{S 4}$ | C | G | A | G | C | T | G | C | T |
|  |  |  |  |  |  |  |  |  |  |



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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | C | G | C | G | C | T | G | T | T |
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| S3 | T | G | A | A | C | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |
|  |  |  |  |  |  |  |  |  |  |



## Resampling methods for assessing the support of a tree given the data

Approach 1 - Jackknife: Remove a random subset of alignment columns and re-compute the tree. Typically a $50 \%$ Jackknife analysis is performed.
repeat
$\mathrm{n}^{*}$ times


## Resampling methods for assessing the support of a tree given the data

Approach 2 - Bootstrap: Resample randomly chosen columns from the original alignment (with replacement) to obtain a new alignment with the same length as the original alignment.

| Taxon | 7 | 7 | 9 | 8 | 5 | 6 | 7 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [s1 | G | G | T | T | C | T | G | C | G |
| S2 | C | C | T | T | C | T | C | C | G |
| S3 ] | G | G | T | C | C | T | G | T | G |
| S4 | G | G | T | C | C | T | G | c | G |

$\left.\begin{array}{cccccccccc}\text { Taxon } & \mathbf{1} & \mathbf{1} & \mathbf{4} & \mathbf{4} & \mathbf{7} & \mathbf{7} & \mathbf{1} & \mathbf{5} & \mathbf{9} \\ \hline \text { S1 } & \text { C } & \text { G } & \text { C } & \text { G } & \text { C } & \text { T } & \text { G } & \text { T } & \text { T } \\ \text { S2 } & \text { C } & \text { G } & \text { C } & \text { A } & \text { C } & \text { T } & \text { C } & \text { T } & \text { T } \\ \text { S3 } \\ \text { S4 }\end{array}\right] \quad$ T

| Taxon | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | C | G | C | G | C | T | G | T | T |
| S2 | C | G | C | A | C | T | C | T | T |
| S3 | T | G | A | A | C | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |


${ }^{*} \mathrm{n}$ is typically 100 or 1000

## Resampling methods for assessing the support of a tree given the data

Approach 2 - Bootstrap: Resample randomly chosen columns from the original alignment (with replacement) to obtain a new alignment with the same length as the original alignment.
$\left.\begin{array}{llllllllll}\text { Taxon } & \mathbf{7} & \mathbf{7} & \mathbf{9} & \mathbf{8} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{1} & \mathbf{2} \\ \hline \text { S1 } & \text { G } & \text { G } & \text { T } & \text { T } & \text { C } & \text { T } & \text { G } & \text { C } & \text { G } \\ \text { S2 } & \text { C } & \text { C } & \text { T } & \text { T } & \text { C } & \text { T } & \text { C } & \text { C } & \text { G } \\ \text { S3 } \\ \text { S4 }\end{array}\right] \quad$ G

| Taxon | 1 | 1 | 4 | 4 | 7 | 7 | 1 | 5 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [ 1 | C | G | C | G | C | T | G | T | T |
| S2 | C | G | C | A | c | T | C | T | T |
| S3] | T | G | A | A | c | T | G | C | T |
| S4 | C | G | A | G | C | T | G | C | T |

s1
**value is typically given in percent

Maximum Parsimony and Maximum Likelihood only evaluate trees and do not reconstruct them! Finding the best tree is highly problematic!

1. Exhaustive Search: evaluates every possible tree and hence an optimal solution is guaranteed. Limit: 10-12 taxa
2. Branch and Bound: excludes parts from the tree space from the search where the optimal tree cannot be found. Guarantees to find the optimal tree.
3. Heuristics: Can be applied to large taxon sets but does not guarantee an optimal solution

Finding the best tree
Evaluate random rearrangements of the starting tree and accept new tree if it improves $P(D \mid M, T)$. Continue until convergence.


Again we have an iterative stochastic process as we have seen in the alignment case

## Tree rearrangements in RAxML*



## Modeling rate across sites

## (Substitution rate heterogeneity across sites)

```
SRYC_DROME/358-380
INSM1_HUMAN/441-464
XFIN_\overline{XENLA/1276-1298}
XFIN_XENLA/1044-1066
ZNF76_HUMAN/285-309
CF2_DROME/401-423
IKZF1_MOUSE/144-166
EVI 1_HUMAN/131-154
TRAl_CAEEL/337-362
SUHW_DROAN/349-373
EGR1_HUMAN/396-418
ADR1_YEAST/104-126
SDC1_CAEEL/268-290
SDC1_CAEEL/145-168
KRUH_DROME/299-321
TTKB_DROME/538-561
KRUP_DROME/222-244
BNC1_HUMAN/928-951
ESCA_DROME/370-392
ADR1_YEAST/132-155
CF2_DROME/429-451
ZG28_XENLA/174-196
ZG3_\overline{XENLA/6-28}
YL57_CAEEL/26-49
ZG5A_XENLA/90-112
ZG52_XENLA/6-27
P43_XENBO/45-69
202_XENLA/34-59
ZG8_XENLA/146-168
SDC\overline{1}_CAEEL/652-674
ZO61_XENLA/62-84
ZG44_XENLA/5-27
```

```
YQCD . . ICG. . .QKFVQKINLTHHARI . . H
HLCP. . .VCG. . .ESFASKGAQERHLRL. .LH
YGCN. . .CCD. . .RSFSTHSASVRHQRM . . C
YKCG . . .LCE . . .RSFVEKSALSRHQRV . . .H
YTCPE.PHCG . . RGFTSATNYKNHVRI . . .H
YTCS . . YCG . . KSFTQSNTLKQHTRI . . . H
FQCN. . .QCG. . . ASFTQKGNLLRHIKL . . .H
YECE. . .NCA. . . KVFTDPSNLQRHIRS . .QH
YSCQI.PQCT. . . KSYTDPSSLRKHIKA. .VH
YACK...ICG. . . KDFTRSYHLKRHQKYS.SC
FACD...ICG...RKFARSDERKRHTKI...H
FVCE. . .VCT. . .RAFARQEHLKRHYRS . . .H
YFCH . . ICG. . .TVFIEQDNLFKHWRL . . . H
YMCQ . . VCL . . TLFGHTYNLFMHWRT . .SC
FECE. . .FCH . . KLFSVKENLQVHRRI . . . H
YPCP. . FCF. . . KEFTRKDNMTAHVKI. .IH
FTCK...ICS. . .RSFGYKHVLQNHERT . . . H
ITCH . . LCQ. . . KTYSNKGTFRAHYKT . .VH
CKCN. . .LCG. . . KAFSRPWLLQGHIRT . . . H
YPCG . . .LCN. . . RCFTRRDLLIRHAQK. .IH
FRCG. . YCG. . .RAFTVKDYLNKHLTT. . .H
FTCT...ECG. . .KCLTRQYQLTEHSYL . . . H
FMCT . . KCG. . . KCLSTKQKLNLHHMT . . . H
YLCY. . YCG. . . KTLSDRLEYQQHMLKK. .VH
FSCT. ..VCG. . EMFTYRAQFSKHMLK. ..H
FTCP...ECG. . . KRF.SQKSNCWHTED . . . H
WKCGK. KDCG . . . KMFARKRQIQKHMKR . . . H
YSCA. . .DCG . . KHFSEKMYLQFHQKNPSEC
FTCT. ..ECG. . EHFANKVSLLGHLKM...H
VVCF. . .HCG. . TRC.HYTLLHDHLDY . .CH
FTCF...ECG. . .TCFVNYSWLMLHIRM...H
FACT . . KCK. . .RRFCSNKELFSHKRI . . .H
```


## Modeling rate across sites

## Revisiting substitution models



We re-scale the substitution rate in a site-specific manner, i.e. the substitution rate at a position / is

$$
Q=\left(\begin{array}{llll}
\mathbf{A} \mathbf{C} & \mathbf{G} & \mathbf{T} \\
- & a & b & c \\
a & - & d & e \\
b & d & - & f \\
c & e & f & -
\end{array}\right)
$$

It is a convention to set the diagonal entries $q_{i i}$ such that the rows sum up to 0 . Thus,

$$
q_{i i}=-\sum_{j \neq i} q_{i j}
$$

However, this model assumes that all sites in a sequence, or all columns in an alignment evolve with the same relative rate. Note, that we can rewrite the total rate for a given position as

$$
q_{i}=\sum_{j \neq i} q_{i j}
$$

We can now introduce a neutral parameter $r=1$ such that can re-write $q_{i}$ as $q_{i}{ }^{*} r$
For a sequence of $L$ characters we have now the possibility to give the parameter $r$ for $I=1$... $L$ a site specific value $r_{r}$.

## Modeling rate across sites <br> Common approaches

Protability Dessify for Guresa Divsitubien wah Veriatio Alpha and Betaet.as


Continuous Gamma distribution with a mean of 1*. Note that the parameter $\alpha$ determines the shape of the distribution.
(Problem of over-parameterization and over-fitting)


Fig. 1. Discrete approximation to the gamma distribution $G(\alpha, \beta)$, with $\alpha=\beta=1 / 2$. Four categories are used to approximate the continuous distribution, with equal probability for each category. The three boundaries are $0.1015,0.4549$, and 1.3233 , which are the percentage points corresponding to $p=1 / 4,2 / 4,3 / 4$. The means of the four categories are $0.0334,0.2519,0.8203,2.8944$. The medians are $0.0247,0.2389,0.7870,2.3535$, and these are scaled to get 0.0291 , $0.2807,0.9248$, and 2.7654 , so that the mean of the discrete distribution is one.

## Modeling rate across sites Common approaches

Protability Dessiry far Gumena Dissitusien wah Veriatio Alpha and Betaetas


Likelihood based tree reconstruction methods assign each position in the alignment either its own relative rate (Gamma model) or assigns it to a given rate category. In the latter case you are asked how many rate categories you want to use (values range typically between 4 and 12).


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## Looking at trees via their splits

Each branch of a tree describes a split of OTUs into two sets
These sets correspond to the two clades associated with the branch e.g. black branch of the tree specifies the split ABCD | EFG -can also be written ADCB \| GFE etc.
-i.e. the taxon lists in the two halves of the split are unordered


## Looking at trees via their splits

Splits are either
trivial

- example: F | ABCDEG
- associated with terminal branches
- provide no information about topology structure non-trivial



## Looking at trees via their splits

Complete list of splits described by a tree allows reconstruction of that tree's topology

Helps to consider the sets of clades described by the splits


## Split Compatibility

Sets (e.g. pairs) of splits are either:

## compatible

- a tree can be drawn that contains all splits in the set
incompatible
-a tree cannot be drawn that contains all splits in the set
Definition: Two splits $\mathrm{W} \mid \mathrm{X}$ and $\mathrm{Y} \mid \mathrm{Z}$ are compatible, i.e. not contradictory, if at least one intersection of $W \cap Y$, $W \cap Z, X \cap Y, X \cap Z$ is empty.

Which of these sets of splits is incompatible?

| $A B \mid C D E$ |
| :--- |
| $D E \mid A B C$ |
| (i) |

BCDFGH|AE ABEGH | CDF BG |ACDEFH
(ii)

Sets of trees can be summarized by looking at their split sets: Strict Consensus Trees


|  | i | ii | iii | iv | v | vi | vii | vii <br> i |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{AB} \mid \mathrm{CDEF}$ | $*$ | $*$ | $*$ | $*$ | $*$ | $*$ | $*$ | $*$ | 8 |
| $\mathrm{CD} \mid \mathrm{ABEF}$ |  | $*$ |  | $*$ |  |  |  |  | 2 |
| $\mathrm{EF} \mid \mathrm{ABCD}$ | $*$ | $*$ | $*$ |  | $*$ |  |  | $*$ | 5 |
| $\mathrm{ABC} \mid \mathrm{DEF}$ | $*$ |  | $*$ |  |  |  |  |  | 2 |
| $\mathrm{DE} \mid \mathrm{ABCF}$ |  |  |  |  |  |  | $*$ |  | 1 |
| $\mathrm{CF} \mid \mathrm{ABED}$ |  |  |  |  |  | $*$ | $*$ |  | 2 |
| $\mathrm{ABD} \mid \mathrm{ECF}$ |  |  |  |  | $*$ | $*$ |  | $*$ | 3 |



## Sets of trees can be summarized by looking at their split sets: 50\% Majority Rule Consensus Trees



## Label the Branches!

Branches of consensus tree labeled to indicate proportion of trees containing that branch/split


